Wave and Circulation Prediction on Unstructured Grids

Joannes J. Westerink
Department of Civil Engineering and Geological Sciences
Department of Mathematics
156 Fitzpatrick Hall of Engineering
University of Notre Dame
Notre Dame, IN 46556
phone: (574) 631-6475 fax: (574) 631-9236 email: jjw@nd.edu

Clint Dawson
Department of Aerospace and Engineering Mechanics
Institute for Computational Engineering and Sciences
The University of Texas at Austin
1 University Station C0200
Austin, TX 78712
phone: (512) 475-8627 fax: (512) 471-8694 email: clint@ices.utexas.edu

Rick A. Luettich
Institute of Marine Sciences
University of North Carolina at Chapel Hill
3431 Arendell Street
Morehead City, NC 28557
phone: (252) 726-6841 ext. 137 fax: (252)726-2426 email: rick_luettich@unc.edu

Award Number: N00014-06-1-0285
http:// www.nd.edu/~coast

LONG TERM GOALS

The long term goal of this research is to significantly advance computational methods for multi-scale flow physics in geometrically and/or hydrodynamically complex oceanic and coastal ocean environments, through refinements in the defined physics, domain definition and computational grid resolution. The particular focus is on improved coupling of wind generated short wave and circulation models within the framework of adaptive, unstructured grid models for oceanic and coastal waters.

OBJECTIVES

The objective of this project is the dynamic coupling of the ADCIRC circulation model and the short wind wave model SWAN using unstructured adaptive computational meshes. In recent experience with coupling the continuous Galerkin based version of ADCIRC with short wind wave models, it has been found that wave transformation zones require high levels of resolution in order to correctly capture the wave radiation stress forcing to the circulation model. However the wave transformation zone tends to shift depending on the direction and period of the waves. Thus *hp*-adaptive discontinuous Galerkin (DG) based solutions are being developed to optimize the application of high resolution zones to correctly capture the wave coupling within these zones without over-resolving adjacent areas.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
30 SEP 2008	**Annual**	**00-00-2008 to 00-00-2008**

4. TITLE AND SUBTITLE **Wave And Circulation Prediction On Unstructured Grids**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **University of Notre Dame,Department of Mathematics, Department of Civil Engineering and Geological Scienc,156 Fitzpatrick Hall of Engineering,Notre Dame,IN,46556**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
code 1 only

14. ABSTRACT
The long term goal of this research is to significantly advance computational methods for multi-scale flow physics in geometrically and/or hydrodynamically complex oceanic and coastal ocean environments, through refinements in the defined physics, domain definition and computational grid resolution. The particular focus is on improved coupling of wind generated short wave and circulation models within the framework of adaptive, unstructured grid models for oceanic and coastal waters.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**14**	

APPROACH

We have focused on three areas to accomplish the defined objectives. The first area is implementing and evaluating *hp*-adaptive schemes for DG solutions to the shallow water equations (SWE) in anticipation of coupling with a mesh adaptive version of the SWAN wave model which is under development. This will improve the unstructured grid capabilities for the ADCIRC circulation model by implementing *hp*-adaptive methods to ensure that sufficient grid resolution is provided for all relevant flow scales. The present implementation of ADCIRC is based on both Continuous Galerkin (CG) and DG finite element methods. Currently we are intensively developing the DG based algorithms. DG algorithms are particularly well-suited for both propagation and advection dominated problems with or without sharp gradients in the forcing function, bathymetry, and/or flow. DG methods inherently preserve mass perfectly on an elemental level, which make them ideal for coupling flow and transport models. Example problems include flows with strong eddies such as those issuing from and through inlets; flows through rapidly varying bathymetry such as canyons or deep scour holes in inlets; and flows with sharp fronts such as tidal bores, ebb tide – waves interactions in inlets and wetting fronts. DG methods are conceptually similar to finite volume methods although DG methods are readily implemented with higher-order bases. DG methods are also ideal since non-conforming *h* and *p* refinement and adaptivity can be implemented. In addition, DG methods are significantly more accurate on a per degree of freedom basis than CG methods. Finally DG based methods require less message passing within a parallel framework and are therefore better suitable for massively parallel processing.

The second area is evaluating the influence of grid resolution in the SWE circulation model in relation to the wind-wave model and the computed wave radiation stress set up.

The third area is redesigning the wave-circulation model interface and integrally coupling these models to maximize the scalability of the combined models. The goal is to remain scalable up to 10,000's of processors.

WORK COMPLETED

Work has continued on the DG and CG ADCIRC SWE modeling system development concentrating on model efficiency, scalability, automated *p*-adaptivity, automated *h*-adaptivity, further improvements in the robustness of wetting/drying, and validation in inlet systems. This work has been focused towards being able to implement *hp* adaptivity within the framework of the coupled wave-current model. In addition, work has rapidly progressed in developing a tightly coupled scalable parallel wave-current model by integrating the PUnSWAN (Parallel Unstructured SWAN) and PADCIRC (Parallel ADCIRC) models to work on the identical unstructured grids. The parallel implementation of UnSWAN uses PADCIRC's highly efficient and scalable parallel MPI based communications subroutines. This synergy allowed for the very rapid development of the parallel version of UnSWAN by Delft. When run in parallel, the partitioned sub-grids for PUnSWAN and PADCIRC are distributed on identical processor. This tight coupling eliminates massive communications issues in that both PUnSWAN and PADCIRC only need communicate along shared sub-grid boundaries and do not need inter-model global or intra-model global communications. In fact, all inter-model communications are entirely local and occur through memory and/or cache. This paradigm is essential in achieving coupled code scalability that extends to 10,000's of processors or computational cores.

DG and CG model efficiency and scalability

The efficiency of the DG solutions to the SWE's has been a matter of intensive study. DG solutions have more degrees of freedom for comparable interpolation orders than CG solutions. In addition, our DG solutions are fully explicit, constraining time steps. We have completed a comparison study of DG and CG methods that finds that for comparable accuracy, both methods have very similar computational costs due to DG's higher accuracy per degree of freedom (Kubatko et al., *Journal of Scientific Computing*, Accepted for Publication). In addition, this study indicates DG solutions are far superior in their scalability compared to CG solutions. Nonetheless, CG solutions are still performing well in their scalability for large computational meshes.

Detailed studies of the time step limitation of the two-dimensional RKDG SWE solutions were performed to examine the stability properties of these solutions (Kubatko et al., *Journal of Computational Physics*, In Press). Semi discrete DG approximations using polynomials spaces of degree $p = 0, 1, 2,$ and 3 were considered and discretized in time using a number of different strong-stability-preserving (SSP) Runge–Kutta time discretization methods. Two structured triangular grid configurations were analyzed for wave propagation in different directions. Approximate relations between the two dimensional CFL conditions were derived and previously established one-dimensional conditions were observed after defining an appropriate triangular grid parameter h and a constant that is dependent on the polynomial degree p of the DG spatial approximation. Numerical results verify the CFL conditions that were obtained, and "optimal", in terms of computational efficiency, two-dimensional RKDG methods of a given order were identified.

DG SWE wetting/drying

Further improvements have been made in the robustness and accuracy of the RKDG SWE based wetting/drying algorithm, including proving convergence for a range of problems (Bunya et al., *Computer Methods in Applied Mechanics and Engineering*, Accepted for Publication). The method takes a fixed mesh approach as opposed to mesh-adaptation techniques and applies a post-processing operator to ensure the positivity of the mean water depth within each finite element. In addition, special treatments were applied in the numerical flux computation to prevent an instability due to negative mass for an arbitrary time step. The proposed wetting and drying treatment was verified through comparisons with five problems with exact solutions and convergence rates were examined. Applying linear interpolation, convergence rate for problems with discontinuities range between 0.8 and 1.0 while for problems with smooth solutions they range from 1.1 to 1.6. The combination of the proposed wetting and drying treatment and a TVB slope limiter was also tested.

h- and p- adaptivity

The DG SWE code *p*- (polynomial order) and standard *h*- (grid) refinement/adaptivity options have been further refined and our progress is summarized by Kubatko et al. (*Computer Methods in Applied Mechanics and Engineering*, Accepted for Publication). The *p*–adaptive algorithm that was implemented dynamically adjusts the order of the elements of an unstructured triangular grid based on a simple measure of the local flow properties of the numerical solution. Time discretization is accomplished using optimal strong-stability preserving (SSP) RK methods. The methods were tested on two idealized problems of coastal ocean modeling interest with complex bathymetry—namely, the idealization of a continental shelf break and a coastal inlet. Numerical results indicate the stability, robustness, and accuracy of the algorithm, and it is shown that the use of dynamic *p*–adaptive grids

offers savings in CPU time relative to grids with elements of a fixed order p that use either local h–refinement or global p–refinement to adequately resolve the solution while offering comparable accuracy. Detailed studies of h-adaptivity and convergence are ongoing looking at resolution requirements within these coastal regimes.

Validation of DG SWE Solution in Coastal Inlets

Detailed validation of the DG SWE code continues focusing in particular on advection dominated inlet problems with extensive wetting/drying on floodplains. We are examining differences between CG and DG solutions within the context of grid resolution.

Integrated Coupling of PUnSWAN and PADCIRC

For both the serial coupling and the parallel coupling, code integration was performed at a low level. Both codes use identical grids and define the variables at the vertices of the triangle elements. UnSWAN was compiled as a subroutine inside ADCIRC. When possible, the same compiler flags were used for the source files of both models. At this low-level, tight coupling makes it possible to pass all inter-model communications through memory in an efficient manner. Because the two models use the same grids, and because the models are running sequentially on the same processor, information can be passed very efficiently from UnSWAN to ADCIRC and vice versa. Water levels, currents and wind speeds are passed from ADCIRC to SWAN, and the gradients of the wave radiation stresses are passed from UnSWAN to ADCIRC.

For the parallel implementation, identical Metis generated sub-grids are assigned to the same processor or core for both PUnSWAN and PADCIRC. Thus the same sub-grid is being used by both models on the same processor, with the 2 codes running sequentially for that specific sub-grid. This means that only intra model sub-grid communications need occur along the shared rows of elements. No inter model communication or global communication need occur. This is the most efficient possible paradigm in terms of maximizing data handling that occurs within cache and/or memory and minimizing the traffic on the network.

The coupled parallel PUnSWAN/PADCIRC model is fully operational and is being extensively verified using sequences of grids with varying degrees of resolution.

RESULTS

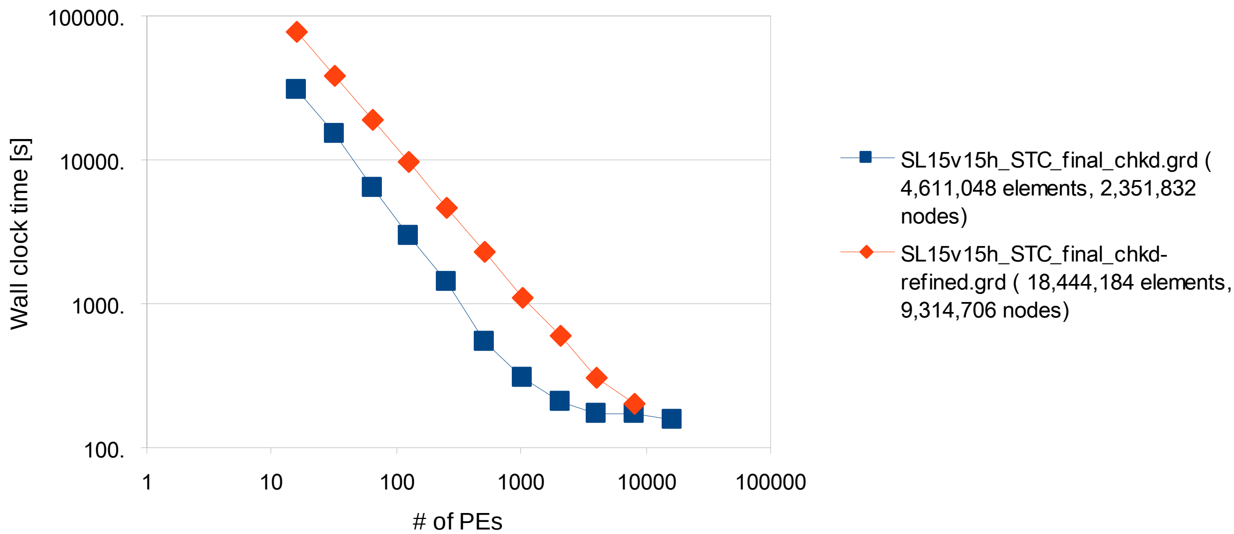
Many of the results from the past year are currently being published in a sequence of four papers which we have included with this report.

Kubatko et al., *Journal of Scientific Computing*, Accepted for Publication.
Kubatko et al., *Journal of Computational Physics*, In Press.
Bunya et al., *Computer Methods in Applied Mechanics and Engineering*, Accepted for Publication.
Kubatko et al., *Computer Methods in Applied Mechanics and Engineering*, Accepted for Publication.

The current results that are ongoing and are not yet in paper form relate to scalability, further validation in inlets and wave-current convergence and performance testing.

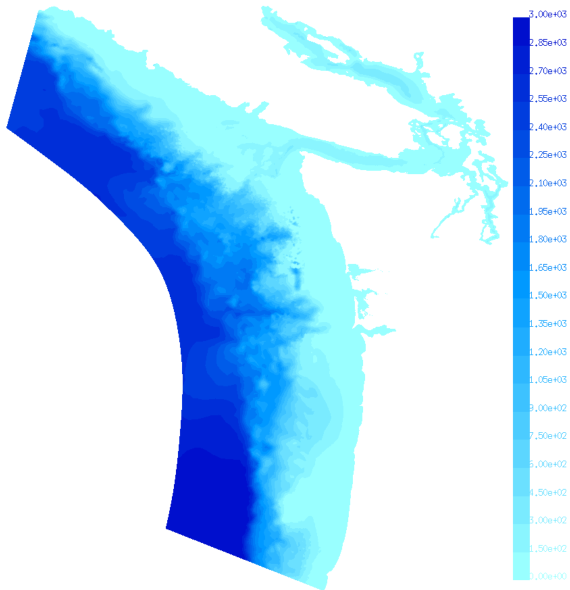
PADCIRC Scalability

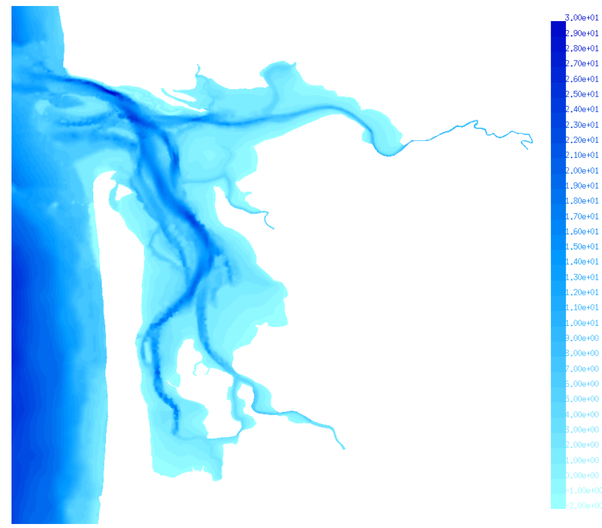
Although the PADCIRC DG code can out perform the CG code in terms of scalability, the initial couplings to the PUnSWAN have been with the CG code. Therefore we have a significant interest in maintaining the scaling and performance of the CG PADCIRC family of algorithms as we transition to DG. We have tested the 2.3 million node large operational grid for Southern Louisiana storm surge modeling as well as a 4x refined version of this grid with 9.3M nodes on Ranger (A SUN Constellation with AMD Opteron quad-core processors for a total of 62,976 computational cores). Using 1 second time steps, the operational grid maintains scalability up to 4096 cores while the large 9.4 million node grid maintains scaling at least up to 8192 cores. We expect that both grids will be able to run at least 48 times faster than real time. It is our goal to maintain this level of performance with the coupled PUnSWAN-PADCIRC model.

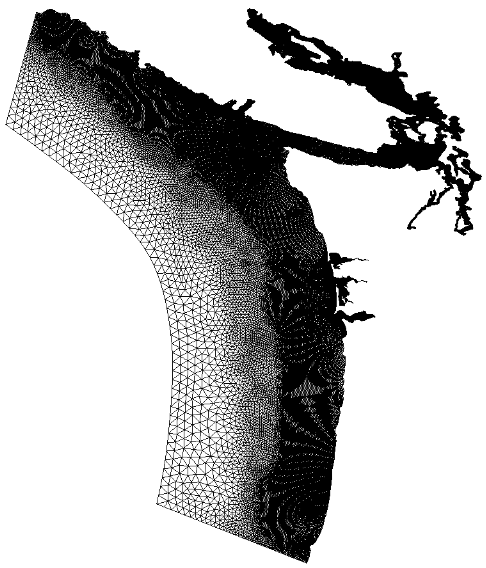


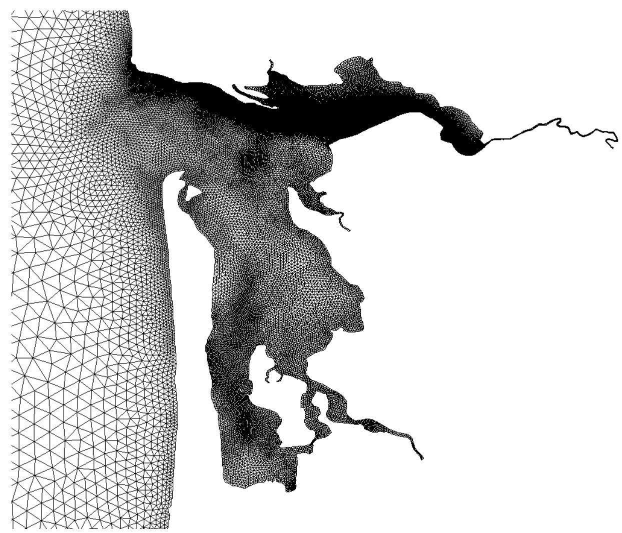
Validation of the DG solution for Inlets

Further detailed studies of tidal elevations and currents in Willapa Bay, WA have been performed with both the CG and DG SWE codes. This area is characterized by its large tidal range, fast currents, strong advection, and large tidal flats. The bathymetry and grid structure are shown below.

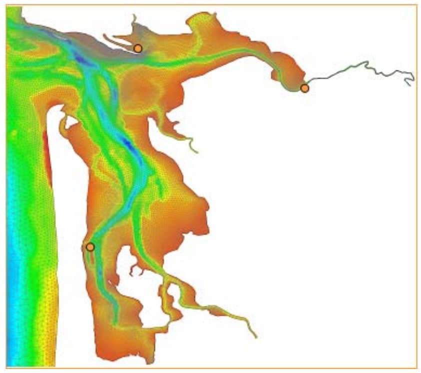




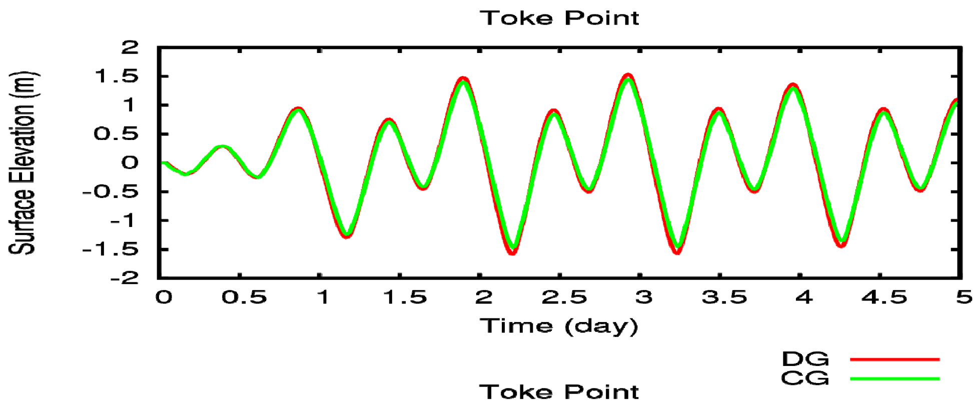


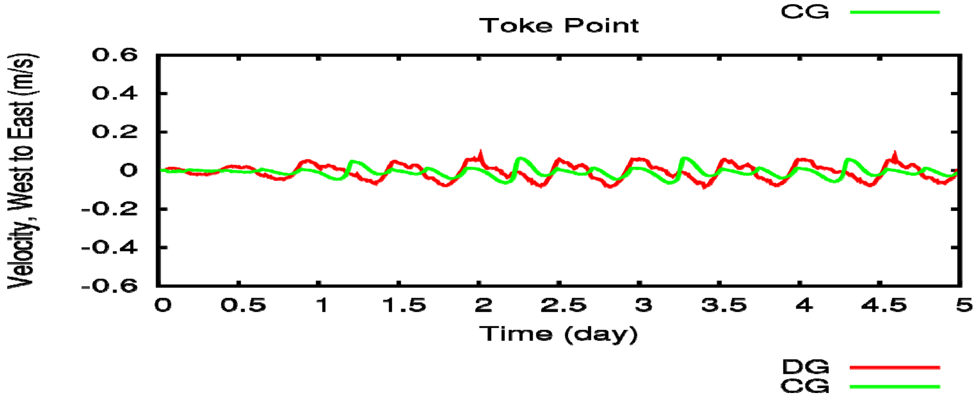


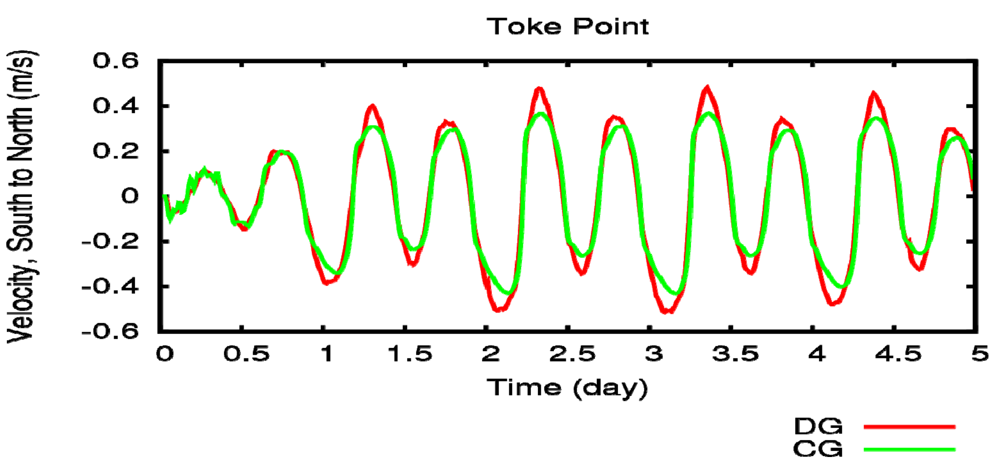
The M_2, S_2, N_2, K_1, O_1, Q_1, P_1, and K_2 tidal constituents are forced using a larger domain model. Comparisons in the tides and currents are made at three stations as shown below.



A sample of the comparisons is shown below.





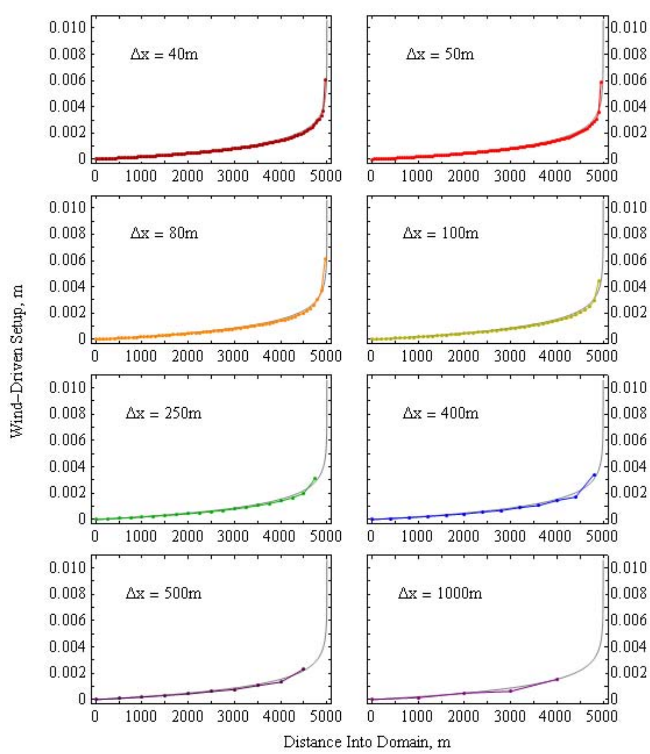


We note that while the elevation solutions are very similar, there are considerable differences in the DG and CG solution in velocities. We are pursuing a systematic grid convergence study to demonstrate that the DG results are superior to the CG results and that additional resolution is needed in the CG solution in order to improve its accuracy. We will quantify error levels for both solutions using Richardson extrapolation and model inter-comparisons at the varying grid resolution levels**.** These cases will also help us quantify required grid resolution levels and help us develop automated sensors for DG based *h* and *p* adaptivity.

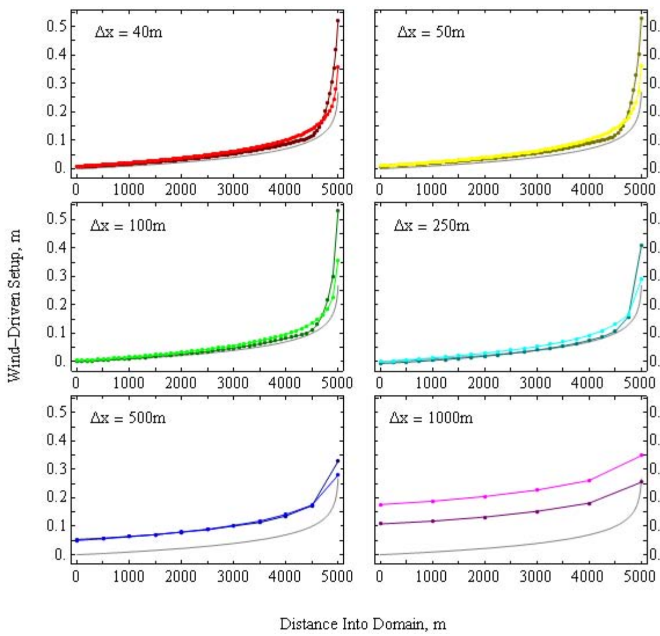
Integrally Coupled Parallel PUnSWAN-PADCIRC Verification

For the coupled parallel PUnSWAN-PADCIRC model we have verified that the physics of wave induced coastal setup is correctly modeled. An analytical solution by Dean and Dalrymple (1984) for wind-driven setup is used to compare the PUnSWAN-PADCIRC model with varying degrees of resolution. We present a sample solution of a test domain with a linear sloping beach with the slope equal to 0.02 and a constant 10m/s wind applied perpendicular to the shore. This linear sloping beach domain has a width of 4km, a length of 6km, bathymetric depth at the open ocean boundary of 100m (below the geoid), topographic elevation at the closed land boundary of -20m (above the geoid). In order to simulate the idealized one dimensional domain, we implemented reflective boundaries in PUnSWAN. Wave energy that would otherwise transmit through a lateral boundary is now reflected back into the domain. The wave energy is also allowed to spread into multiple reflected directions if appropriate.

Steady state (after 2 days) wind plus wave induced set up for 8 meshes are compared to the analytical solution below, indicating a converging solution as resolution is improved.

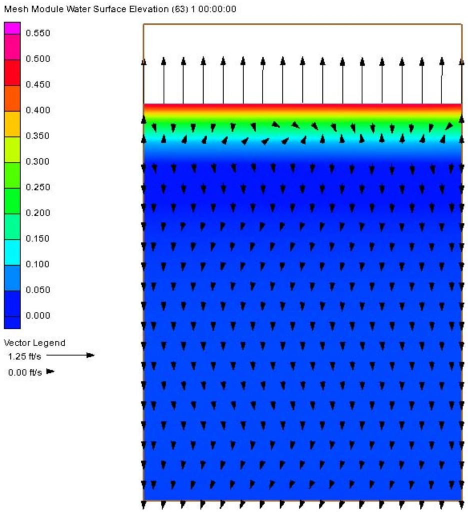


A second test case identical to the previous one, except the wind speed was increased from 10m/s to 50m/s and the linear sloping beach has an increased slope equal to 0.2. Thus, the depth at the open ocean boundary becomes 1,000m below the geoid, and the depth at the closed boundary becomes 200m above the geoid. Wind only (lighter colors) plus wind and wave induced set up (darker colors) for 6 meshes are compared to the analytical solution below, showing that the solutions are converging as resolution is improved and indicating that in this case, the wave induced portion of setup is at least 40% of the total setup.

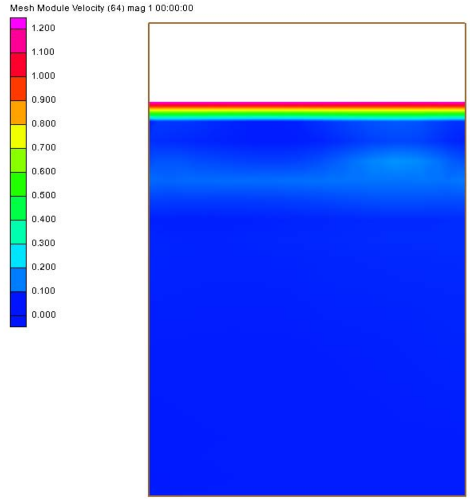


We note that we do not exactly match the analytical solution most likely due to the approximation made for the friction coefficient in the analytical solution and the exact form of the air-sea drag laws used. We note that coarse grids entirely miss the solution with problems in the radiation boundary conditions as well as inverting the coastal setdown/setup effect of the wind driven setup.

A third problem examines the same mild slope domain (0.02) with wave forcing equal to 10 m significant wave that was specified at the south boundary and allowed to enter and break inside the domain. With mesh spacing equal to 250 m, shown below, we note a coastal setup equal to 0.48m and significant velocity errors at the very shallow wet/dry interface. These erroneous currents are up to 1.2 m/s at the shoreline (wet/dry interface) and in fact show up away from the coast as well being as large as 10 cm/sec.

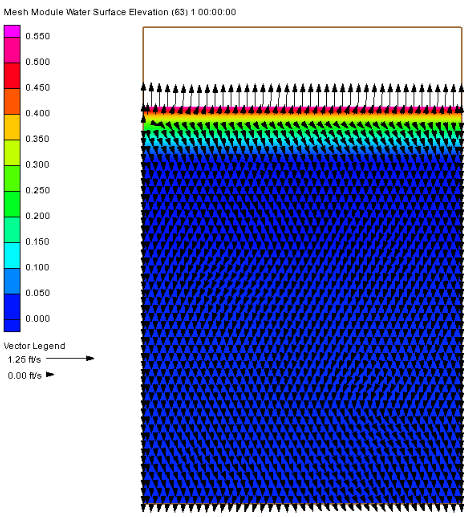


Surface Elevation (m) and velocity vectors (m/s) for 250m mesh

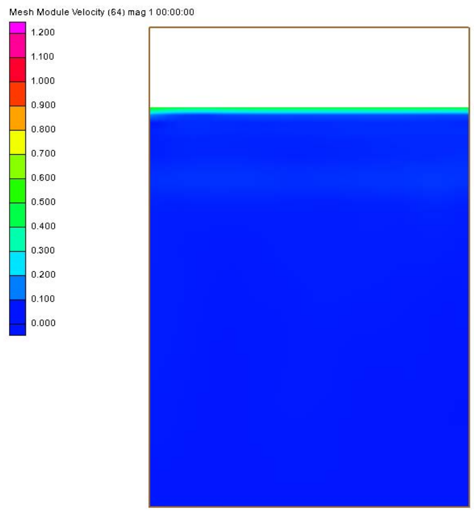


Velocity magnitudes for 250m mesh.

With mesh spacing equal to 100 m, shown below, we note a coastal setup equal to 0.53m and reduced velocity errors at the very shallow wet/dry interface. These erroneous currents are now 0.5 m/s at the shoreline and are also reduced away from the coast to less than 2 cm/sec.

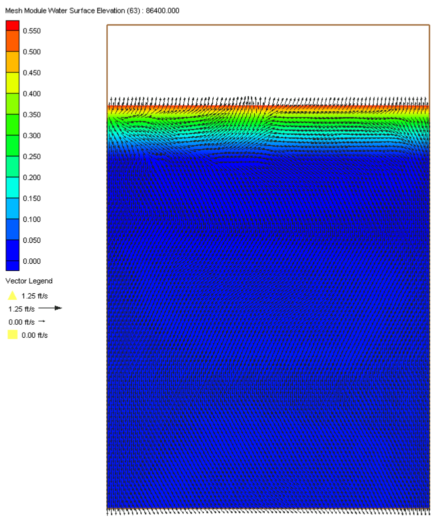


Surface Elevation (m) and velocity vectors (m/s) for 100m mesh

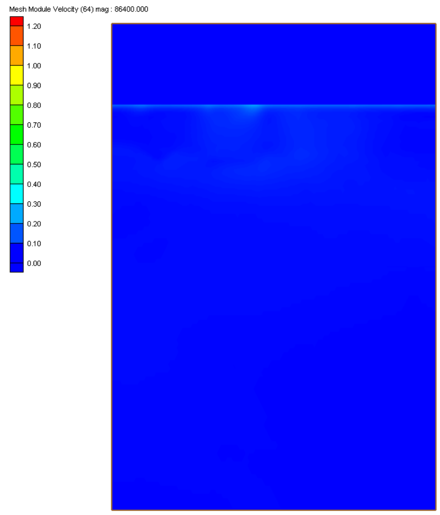


Velocity magnitudes for 100m mesh.

With mesh spacing equal to 50 m, shown below, we note a coastal setup equal to 0.53m and reduced velocity errors at the very shallow wet/dry interface. These erroneous currents are up to 0.2m/s at the shoreline and are also reduced away from the coast to less than 1 cm/sec.



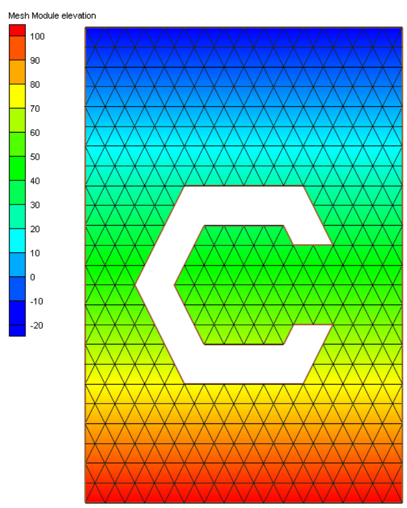
Surface Elevation (m) and velocity vectors (m/s) for 100m mesh



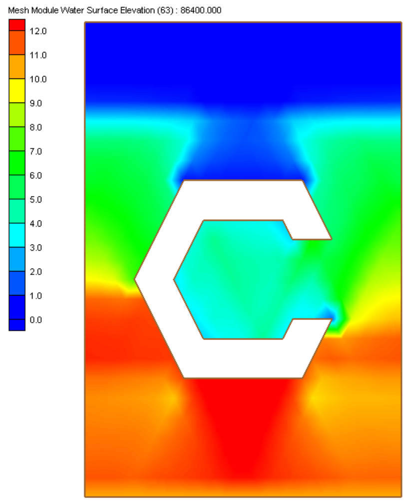
Velocity magnitudes for 100m mesh.

These errors near the breaking zone and the wetting front are related to the level of resolution. It is in these zones where very high gradients in the wave radiation stress and surface elevation exist. This underscores the importance of localized resolution implemented in an unstructured grid. In particular the automated *h* and *p* adaptivity will optimally allow us to ensure accuracy with the minimum computational cost. In addition, we are evaluating the application of wave radiation stress gradients right at the wet/dry interface nodes.

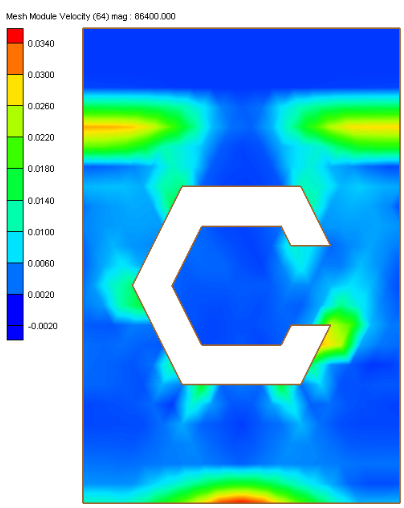
Finally we examine a problem that defines a complex shaped island in the interior of the domain. A modified version of the idealized test case was developed, in which a "C" was carved into the center of the domain, as shown below.

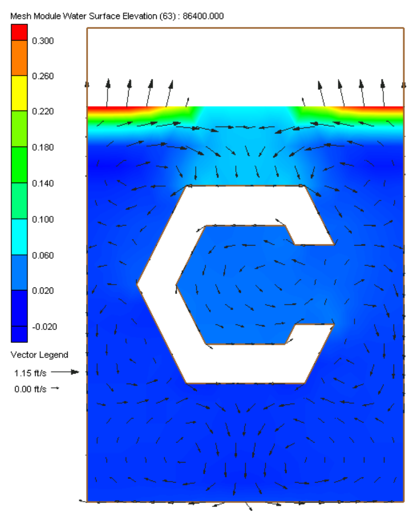


The internal boundary is two elements in width, and exists in depths ranging from 20 m to 70 m. With the exception of the new, internal boundary, all other simulation parameters remained the same. The significant wave heights and wave radiation stresses in this modified test case are shown below.



The waves collect against the southern part of the "C" and grow to 12 m. Along the west side, they flow through an admittedly under-resolved channel and then break on the north side. Along the east side, they bend and reflect into the interior of the "C." Note the reflection of the waves inside the "C" itself, where a green line depicts waves bouncing two or three times inside the internal boundary. Finally, the waves that do reach the north end of the domain are dissipated due to depth-limited breaking. The associated gradients in the wave radiation stresses and the water levels and currents are shown below.





About 0.3 m of wave-driven set-up is build along the coastline. This test problem demonstrates the coupled parallel model's ability to collect waves against internal boundaries, reflect them around sharp corners, and generate set-up behind them. Even though the grid is under-resolved, it is a good example of the capability of the coupled parallel model. We are going through a sequence of higher resolved versions of this grid to prove convergence and quantify errors.

Ongoing development

We will complete the grid convergence/validation studies for the DG SWE application in Grays Harbor and Willapa Bay. We will also complete the comprehensive grid convergence/verification of the PUnSWAN-PADCIRC computations. This will include developing a sound understanding of how much grid resolution needs to be applied in typical scenarios and the associated error levels for waves, still water elevations and currents. We will include idealized one dimensional solutions as well as oblique cases with long shore currents.

We will examine the improvements in the associated physics of wave-current coupling by applying the PUnSWAN-PADCIRC model to study hurricane induced storm surge and currents in and around Southern Louisiana using the 2.4 and 9.3 million node grids. We will in particular investigate the improvements in the computed wave fields and wave induced set up delivered by applying a tightly coupled wave-current modeling system as opposed to using a sequence of nested wave models loosely coupled to ADCIRC. In particular, we believe that the improved directionality of the new system will produce significant improvements in wave set up east of the Mississippi River where the protruding delta catches easterly winds, waves and surge. We will also examine the effect of wave modifications on bottom stress. Running the two versions (2.4 and 9.3 million node) of the unstructured grid will allow us to estimate errors using Richardson extrapolation.

We will also study the scalability of the coupled PUnSWAN-PADCIRC model. It is our goal for the combined model to retain the scalability of the PADCIRC codes. We will also investigate running the two models on the same processor using different cores so that they can be run synchronously while still sharing memory, further improving wall clock times. This will still retain the advantages of running on identical sub-grids and can be implemented on multi-core processors.

Finally we will implement a fully coupled PUnSWAN-PADCIRC within the DG framework and apply *h*-adaptivity so that grid accuracy can be optimized for the wave-current computations.

All these ongoing efforts will be published as a sequence of 5 additional journal papers.

IMPACT/APPLICATIONS

This work will significantly improve the accuracy of the computed physics for coastal ocean wave and circulation computations. The physics will be directly improved through the much tighter wave – current model coupling, as well as through the ability to apply much higher resolution grids which will in fact refine themselves to the appropriate levels though the DG adaptivity. This work will also reduce model development times since base meshes developed by hand only need to represent the geometry and not the hydrodynamics. The grids will adjust themselves automatically and optimally to account for the waves and currents as the hydrodynamics evolves in the computation.

REFERENCES

Dean, R.G., .A. Dalrymple. ***Water Wave Mechanics for Engineers and Scientists***. Prentice Hall Inc., New Jersey, 1984.

PUBLICATIONS

Bunya, S., E.J. Kubatko, J.J. Westerink, C. Dawson, "A Wetting and Drying Treatment for the Runge-Kutta Discontinuous Galerkin Solution to the Shallow Water Equations," ***Computer Methods in Applied Mechanics and Engineering***, Accepted for Publication, 2008.

Kubatko, E.J., C. Dawson, J.J. Westerink, "Time Step Restrictions for Runge-Kutta Discontinuous Galerkin Methods on Triangular Grids," ***Journal Computational Physics***, In Press, 2008.

Kubatko, E.J., S. Bunya, C. Dawson, J.J. Westerink, C. Mirabito, "A Comparison of Continuous and Discontinuous Finite Element Shallow Water Models," ***Journal of Scientific Computing***, Accepted for Publication, 2008.

Kubatko, E.J., S. Bunya, C. Dawson, J.J. Westerink, "Dynamic *p*-adaptive Runge-Kutta Discontinuous Galerkin Methods for the Shallow Water Equations," ***Computer Methods in Applied Mechanics and Engineering***, Accepted for Publication, 2008.